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MODIFIED KORTEWEG-de VRIES SOLITONS ON DUST ION ACOUSTIC SOLITARY WAVES IN A WARM PLASMA WITH ELECTRONS' DRIFT MOTION

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Abstract

In this investigation, only compressive solitons are seen to exist in a dusty plasma consisting of positive ions, electrons and immobile negatively charged dust grains in presence of electrons' drift velocity (v'_e). The compressive mKdV solitons of small amplitude exist in a small region $0 \leq v'_e < 3$. The amplitude becomes higher for smaller values of $\sigma = \frac{T_i}{T_e}$ (= ion to electron temperature ratio).

1. Introduction

Several authors have studied the propagation of ion acoustic solitary

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waves in dusty plasmas both theoretically [1-16] and experimentally [17, 18]. The occurrence of dusty plasma in cometary tails, asteroid zones, interstellar clouds, planetary rings, earth's ionosphere and magnetosphere [19-28] make the subject attractive. The wave in dusty plasma is different from that of ordinary plasma and the presence of dust charge in plasma influences the plasma properties leading to new significant results. The waves in dusty plasma is different from that of ordinary plasma and the presence of dust charge in plasma influences the plasma properties leading to new significant results and generating many new problems to study [20, 35-37]. The dust ion acoustic waves have been observed in laboratory experiments [29, 30]. By using reductive perturbative method, Rao et al. [31] have first reported the existence of the dust acoustic waves for low frequency in unmagnetized dust plasma and experimentally verified in a laboratory by Barkan et al. [17]. Ikezi [38] in his investigations predicted that dust particles in plasma could acquire enough charge to produce a large (ratio of electrostatic to kinetic energy) which results in making a strongly coupled plasma. The ability to image dusty plasmas at the particle level allowed many researchers [39, 40] to study the melting phase transition, phonons and other condensed matter phenomena with unprecedented directness.

In all the above investigation, the initial drift motion of the mobile electrons is not considered. However, it is found that the characters of the dust ion acoustic waves are considerably affected with the introduction of electron inertia and its initial drift motion. Kalita and Das [32] have made a comparative study of mKdV and KdV solitons in a cold plasma with negative ions for different values of electrons' drift velocity and mass ratio $Q' \left(= \frac{m_j}{m_i}, \text{ negative to positive ion mass ratio} \right)$. Singh and Bora [41] have studied the instabilities of ion acoustic waves in a dusty plasma by considering electron-drift, collisional, and dust charge fluctuation effects. In their investigation, they found that electron thermal conductivity and charged grains concentration enhance the growth of the ion acoustic mode whereas ion- viscosity, ion-thermal conductivity, and dust charge fluctuations have a stabilizing effect. Kalita and Das [33] have investigated the existence of KdV and modified KdV solitons in a plasma with negative ions and drifting effect of electrons. Liu and Du [42] have investigated the instability of the dust-acoustic waves driven by drifting electrons and ions in a dusty plasma through kinetic theory. Das and Karmakar [34] have studied the formation of dust ion acoustic solitons in a plasma with the electrons' drift velocity through the mKdV equation.

In this paper, we investigate the drifting effect of electrons in the formation of ion-acoustic solitons in a collisionless dusty plasma including electron inertia.

2. Basic Equation and Derivation of mKdV Equation

We consider an unmagnetized dusty plasma consisting of positive ions, electrons and immobile negatively charged dust grains. The fluid equations of motion, governing the collisionless dusty plasma in one dimension are:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x}\right) v_i + \frac{\sigma}{n_i}\frac{\partial p_i}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x}\right) v_i + 3 p_i \frac{\partial v_i}{\partial x} = 0, \tag{3}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0, \tag{4}$$

$$\left(\frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x}\right) v_e = \frac{1}{Q}\left(\frac{\partial \phi}{\partial x} - \frac{1}{n_e}\frac{\partial p_e}{\partial x}\right) = 0, \tag{5}$$

$$\left(\frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x}\right) v_e + 3 p_e \frac{\partial v_e}{\partial x} = 0, \tag{6}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \alpha n_e - n_i + 1 - \alpha, \tag{7}$$

where i and e stand for positive ion and electron, respectively, $Q = \frac{m_e}{m_i}$ (= electron to ion mass ratio), $\sigma = \frac{T_i}{T_e}$ (= ion to electron temperature ratio) and $\alpha = \frac{n_{e0}}{n_{i0}} = 1 - \frac{z_d n_{d0}}{n_{i0}}$ (z_d is the number of elementary charges residing on the dust grain).

We have normalized densities n_i and n_e by the unperturbed densities n_{e0}, ion pressures by characteristic ion pressure $k_b n_{e0} T_e$, time t by the inverse of the characteristic ion plasma frequency, i.e., $\omega_{pi}^{-1} = \left(\frac{m_i}{4\pi n_{e0} e^2}\right)^{\frac{1}{2}}$, distance x by the electron Debye length $\lambda_{De} = \left(\frac{k_b T_e}{4\pi n_{e0} e^2}\right)^{\frac{1}{2}}$, velocities by the ion-acoustic speed $C_s = \left(\frac{k_b T_e}{m_i}\right)^{\frac{1}{2}}$, and the potential ϕ by $\frac{k_b T_e}{e}$; k_b is the Boltzmann constant.

Introducing the stretched variables

$$\xi = \varepsilon(x - Ut), \quad \tau = \varepsilon^3 t \tag{8}$$

with phase velocity U of the wave and using the following expansions of the flow variables in terms of the smallness parameter ε:

$$n_i = 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \cdots,$$

$$n_e = 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \cdots,$$

$$v_i = \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \cdots,$$

$$v_e = v_e' + \varepsilon v_{e1} + \varepsilon^2 v_{e2} + \cdots,$$

$$p_i = p_{i0} + \varepsilon p_{i1} + \varepsilon^2 p_{i2} + \cdots,$$

$$p_e = p_{e0} + \varepsilon p_{e1} + \varepsilon^2 p_{e2} + \cdots,$$

$$\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \cdots. \tag{9}$$

(1)-(7) can be simplified to give the expression for the phase velocity and the KdV equation as follows:

$$\frac{\alpha}{3 - Q(U - v'_e)^2} = \frac{1}{U^2 - 3\sigma}. \tag{10}$$

This gives

$$U = \frac{-y \pm \sqrt{y^2 - 4xz}}{2x}, \tag{11}$$

where

$$x = \alpha + Q, \quad y = -2v'_e Q, \quad z = Qv'^2_e - 3(\alpha\sigma + 1) \tag{12}$$

and the KdV equation as

$$\frac{\partial\phi_1}{\partial\tau} + p\phi_1\frac{\partial\phi_1}{\partial\xi} + q\frac{\partial^3\phi_1}{\partial\xi^3} = 0, \tag{13}$$

where $p = \frac{A}{B}$ and $q = \frac{1}{B}$ with

$$A = \frac{3(U^2 + \sigma)}{(U^2 - 3\sigma)^3} + \frac{3\alpha\{Q(U - v'_e)^2 + 1\}}{\{3 - Q(U - v'_e)^2\}^3}$$

and

$$B = \frac{2U}{(U^2 - 3\sigma)^2} + \frac{2\alpha Q(U - v'_e)}{\{3 - Q(U - v'_e)^2\}^2}.$$

For higher order nonlinearity, we set the nonlinear coefficient of (13) to zero, which gives the critical density. Consequently, we can determine the modified KdV equation with cubic nonlinearity, which is expected to describe the dust ion-acoustic wave at the critical density α_c, given by

$$\frac{U^2 + \sigma}{\{U^2 - 3\sigma\}^3} + \frac{\alpha\{Q(U - v'_e)^2 + 1\}}{\{3 - Q(U - v'_e)^2\}^3} = 0 \tag{14}$$

$$\Rightarrow \alpha_c = -\frac{\dfrac{U^2 + \sigma}{(U^2 - 3\sigma)^3}}{\dfrac{Q(U - v'_e)^2 + 1}{\{3 - Q(U - v'_e)^2\}^3}}. \tag{15}$$

The modified KdV equation is applicable only at the critical density α_c. To derive the modified KdV equation, we consider the stretched variables

$$\xi = \varepsilon(x - Ut), \quad \tau = \varepsilon^3 t. \tag{16}$$

From (1)-(7), (9), (14) and (16), the modified KdV equation can be written as

$$\frac{\partial\phi_1}{\partial\tau} + p'\phi_1^2\frac{\partial\phi_1}{\partial\xi} + q'\frac{\partial^3\phi_1}{\partial\xi^3} = 0, \tag{17}$$

where $p' = \frac{A'}{B'}$ and $q' = \frac{1}{B'}$ with

$$A' = \frac{3(U^4 + 30\sigma U^2 + 9\sigma^2)}{2(U^2 - 3\sigma)^5} - \frac{3\alpha\{5Q^2(U - v'_e)^4 + 30Q(U - v'_e)^2 + 9\}}{2\{3 - Q(U - v'_e)^2\}^5}$$

and

$$B' = \frac{2U}{(U^2 - 3\sigma)^2} + \frac{2\alpha Q(U - v'_e)}{\{3 - Q(U - v'_e)^2\}^2}.$$

3. Condition for Existence of Solution and Solitary Wave Solution

From the expression given in (10), we observe that U is real if

$$v'^2_e \le \frac{3(\alpha + Q)(\alpha\sigma + 1)}{\alpha Q}.$$

The soliton solution of the mKdV equation (17) is possible if A and B are positive and finite for which we find that

$$Q(U - v_e')^2 \neq 3 \text{ and}$$

$$5U^4 + 30\sigma U^2 + 9\sigma^2 > \alpha^6\{5Q^2(U - v_e')^4 + 30Q(U - v_e')^2 + 9\}.$$

To find solitary wave solution of the mKdV equation (17), we introduce the variable $\chi = \eta - V\tau$, where V is the soliton speed in the linear χ-space. Using the boundary condition $\phi_1 = \frac{\partial\phi_1}{\partial\chi} = \frac{\partial^2\phi_1}{\partial\chi^2} = 0$ as $|\chi| \to \infty$, equation (17) can be integrated to give

$$\phi_1 = \sqrt{\frac{6V}{p}}\operatorname{sech}\left(\sqrt{\frac{V}{q}}\eta\right).$$

The amplitude and the width of the solitary waves are given, respectively, by $\phi_0 = \sqrt{\frac{6V}{p}}$ and $\Delta = \sqrt{\frac{q}{V}}$.

4. Results and Discussion

In this unmagnetized dusty plasma under variable pressure with the drifting effect of the electrons, only compressive mKdV solitons of small amplitude are seen to exist. Investigation is made on how the electrons drift velocity affect in the formation of mKdV solitons. It is found that the compressive mKdV solitons of small amplitude exists in a small region $0 \leq v_e' < 3$. Figure 1 shows the amplitudes with variation in α for fixed $V = 0.02$ and $\sigma = 0.05$ for different values of $v_e' = 1.95(1)$, $2.00(2)$, $2.05(3)$. It further reveals that though there is imperceptible variation in amplitude for a large region of α, it becomes prominent in the higher range and the amplitude increases with increase in α. The amplitude Figure 2 of the fast compressive solitons are found to decrease with α for fixed $V = 0.01$ and $v_e' = 0.05$ for different values of $\sigma = 0.05, 0.10, 0.15$. The amplitude

(Figure 3(a)) of the fast compressive mKdV solitons is seen to remain almost constant throughout the lower range of v'_e but decrease very slowly in the narrow upper range of $v'_e(< 1)$ for fixed $V = 0.05$ and $\alpha = 0.30$ for different values of $\sigma = 0.05(1),\ 0.15(2),\ 0.25(3)$. The corresponding widths of the compressive solitons exhibit their linear increase (Figure 3(b)). It is seen from Figure 4(a) that the amplitude of the mKdV solitons decrease with σ for fixed $V = 0.02$ and $v'_e = 1$ for different values of $\alpha = 0.1(1)$, $0.2(2),\ 0.3(3)$. The corresponding widths (Figure 4(b)) of the compressive soliton decreases linearly and very slowly with σ. The amplitude (Figure 5(a)) of the mKdV solitons decrease slowly with v'_e for fixed $V = 0.01$ and $\alpha = 0.10$ for different values of $\sigma = 0.05(1),\ 0.10(2),\ 0.15(3)$. But the corresponding widths (Figure 5(b)) of the mKdV solitons increase linearly with σ.

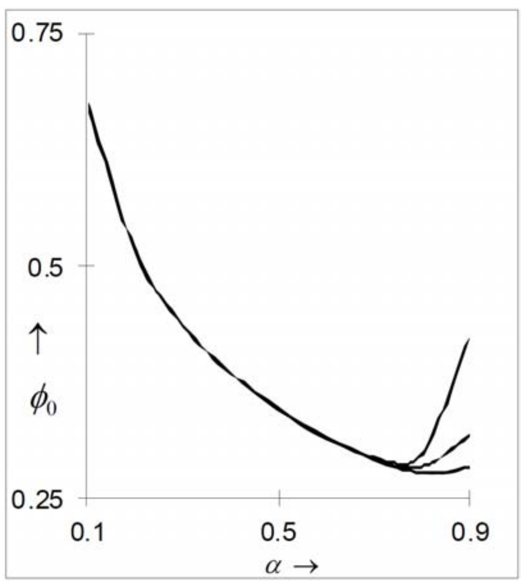

Figure 1. Amplitudes of fast compressive mKdV solitons versus α for fixed $V = 0.02$ and $\sigma = 0.05$ for different values of $v'_e = 1.95(1),\ 2.00(2),$ $2.05(3)$.

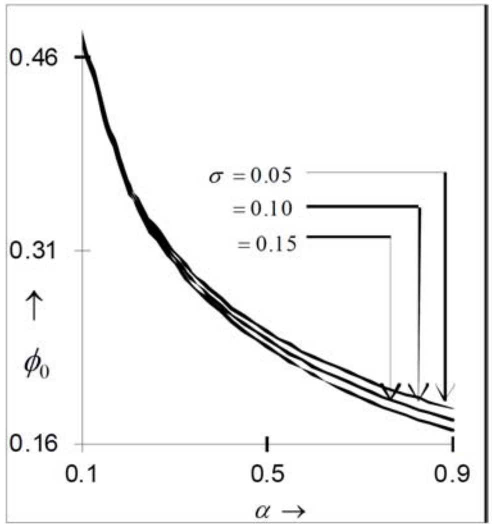

Figure 2. Amplitudes of fast compressive mKdV solitons versus α for fixed $V = 0.01$ and $v'_e = 0.05$ for different values of σ = 0.05, 0.10, 0.15.

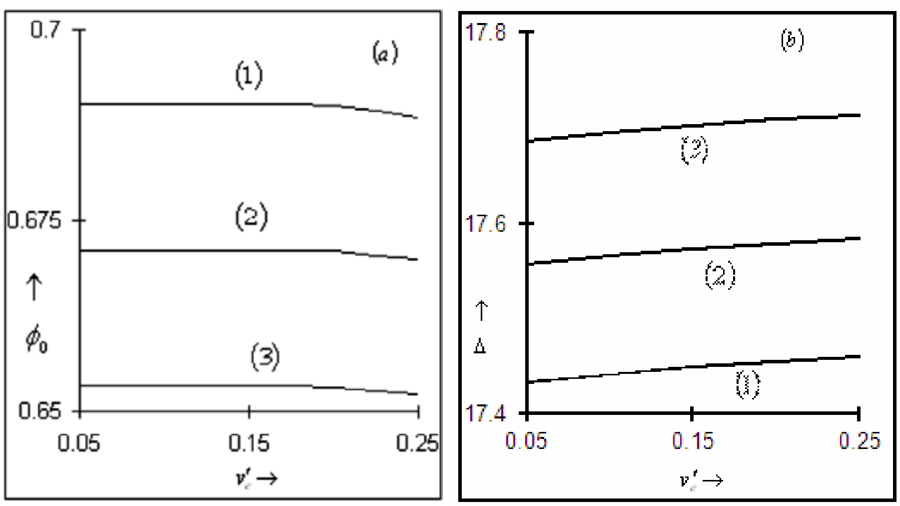

Figure 3. Amplitudes (a) and widths (b) of fast compressive mKdV solitons versus v'_e for fixed $V = 0.05$ and $\alpha = 0.30$ for different values of σ = 0.05(1), 0.15(2), 0.25(3).

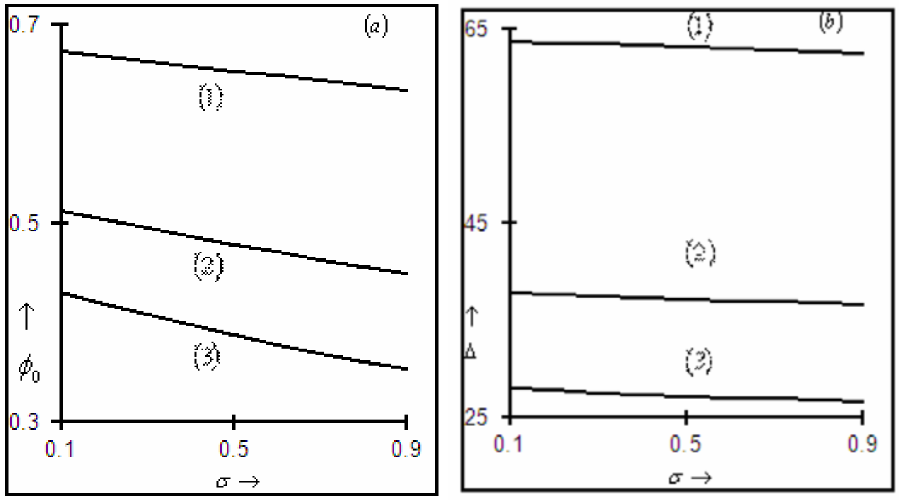

Figure 4. Amplitudes (a) and widths (b) of fast compressive mKdV solitons versus σ for fixed $V = 0.02$ and $\nu_e' = 1$ for different values of $\alpha = 0.1(1)$, 0.2(2), 0.3(3).

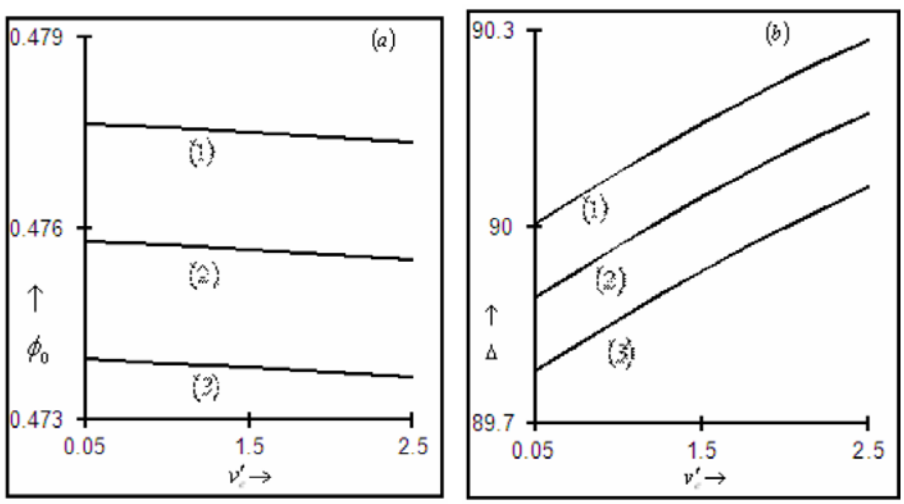

Figure 5. Amplitudes (a) and widths (b) of fast compressive mKdV solitons versus ν_e' for fixed $V = 0.01$ and $\alpha = 0.10$ for different values of σ = 0.05(1), 0.10(2), 0.15(3).

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